성단 역학과
블랙홀 쌍성의 형성

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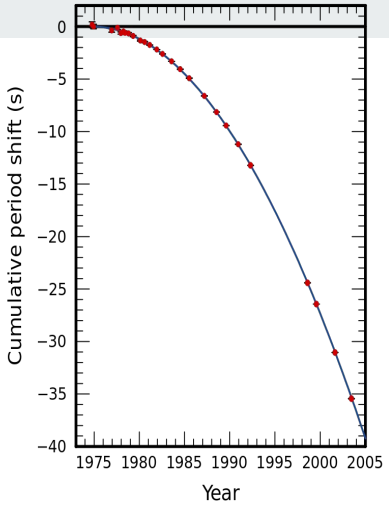
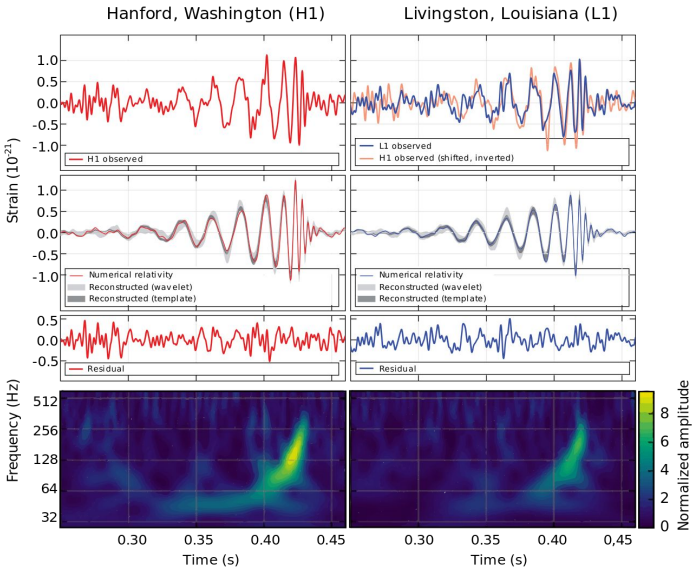
2022 수치상대론 및 중력파 겨울학교
2022.01.19. 서울가든호텔

Introduction



https://en.wikipedia.org/wiki/Hulse%E2%80%93Taylor_binary#/media/File:PSR_B1913+16_period_shift_graph.svg

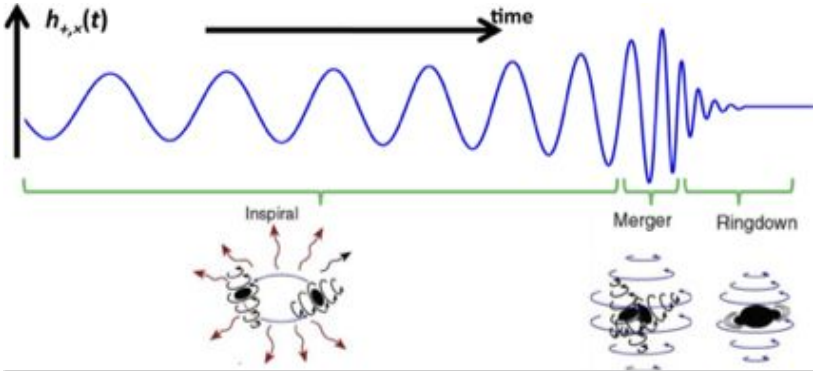
- Gravitational-Waves (GWs)
  - Ripples of spacetime curvature that propagate as waves
  - Indirect detection - PSR 1913+16 (Hulse & Taylor 1974, Weisberg & Taylor 2005)
  - Direct detection - GW150914 by aLIGO
  - Multi-messenger astronomy



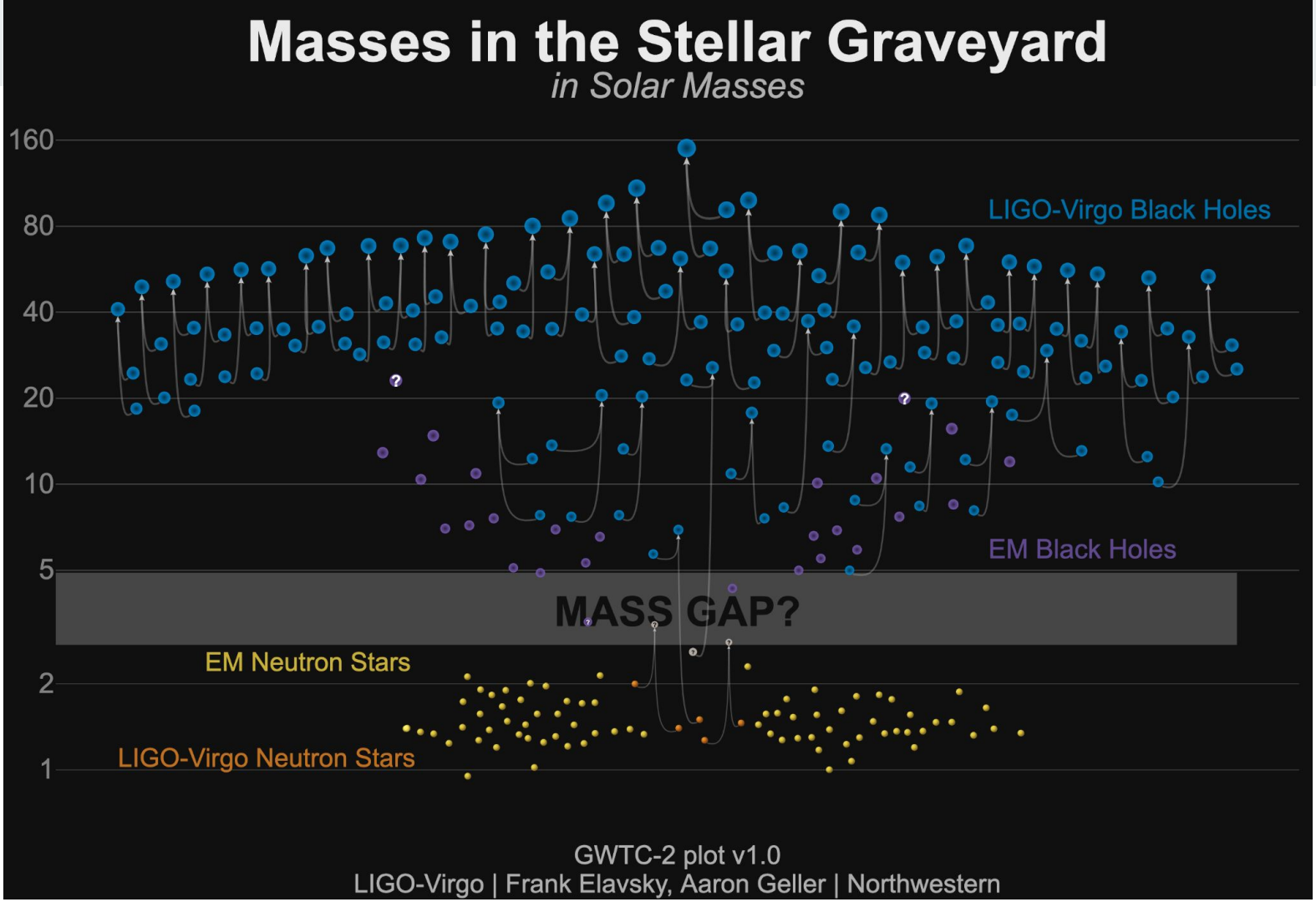
https://en.wikipedia.org/wiki/First_observation_of_gravitational_waves#/media/File:LIGO_measurement_of_gravitational_waves.svg

GW sources

- GRB/Supernova, Spinning Neutron star, Cosmological sources, …
- Compact Binary Coalescence (CBC)
  - Black hole, Neutron star
  - Strong signal
  - Detectable frequency for current GW detectors
  - Binary black hole (BBH)
    - Predictable wave forms
    - Inspiral-Merger-Ringdown
  - About 90 sources are detected.



Inspiral-Merger-Ringdown (M. Favata, SXS, K. Thorne)

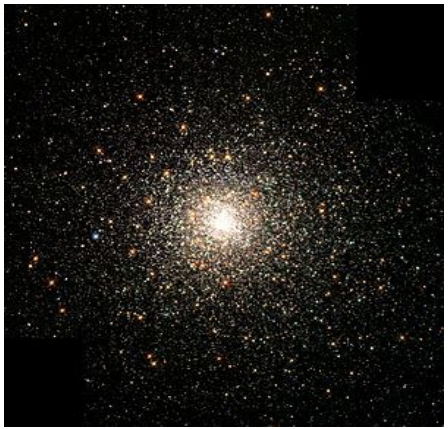

Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
MASS GAP?
EM Neutron Stars
LIGO-Virgo Neutron Stars
GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

Formation of binary BH

- Field
  - Evolution of binary star
- Cluster
  - Dynamical formation

NGC 4414

The Hubble Heritage Team (AURA/STScI/NASA) NASA Headquarters - Greatest Images of NASA (NASA-HQ-GRIN) - http://nix.larc.nasa.gov/info;jsessionid=1sl2so6lc9mab?id=GPN-2000-000933&orgid=12 http://imgsrc.hubblesite.org/hu/db/images/hs-1999-25-a-full_tif.tif



M80 (NGC 6093)

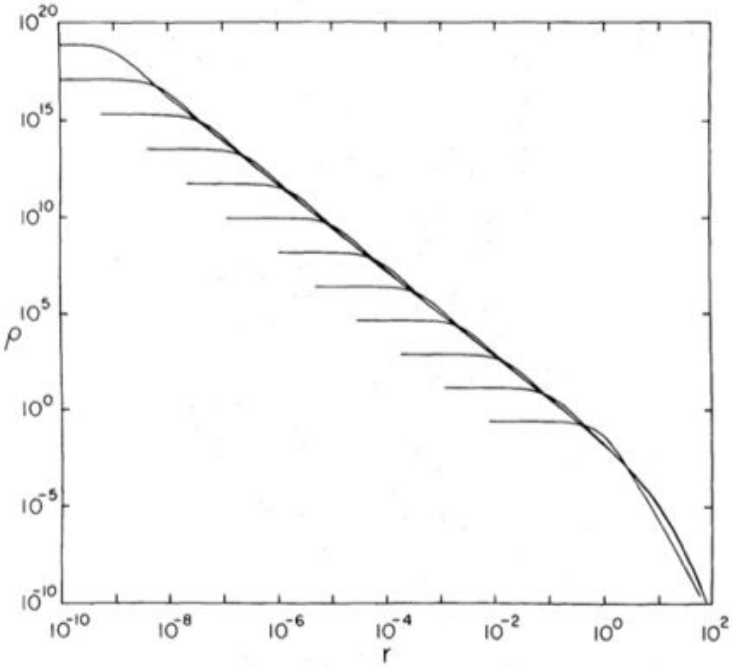
NASA, The Hubble Heritage Team, STScI, AURA - Great Images in NASA Description

Nuclear star cluster of Milky Way

Stefan Gillessen, Reinhard Genzel, Frank Eisenhauer - http://www.eso.org/public/outreach/press-rel/pr-2008/pr-46-08.html

Dynamical evolution of cluster

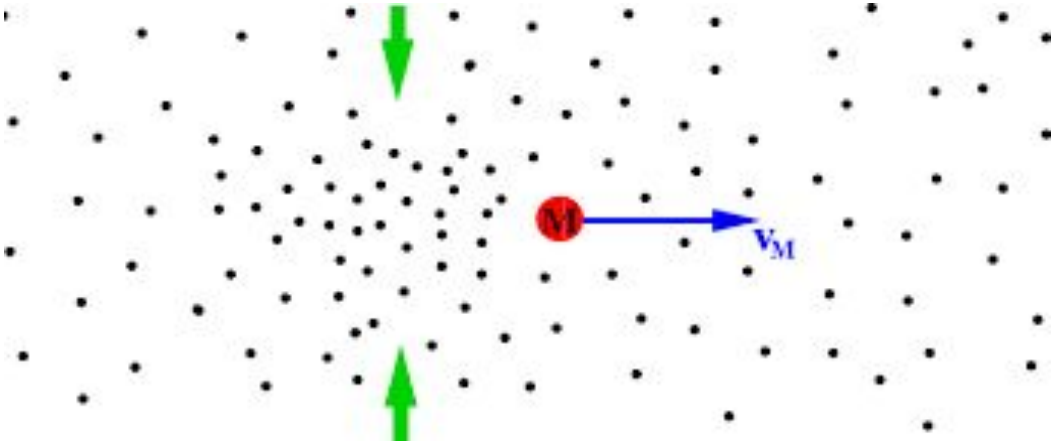
- Evaporation
  - Escape of stars through the relaxation
  - Mass decrease, density increase
- Core collapse
  - Gravothermal instability
  - Negative heat capacity of self-gravitating system
  - Dense core - more encounters
  - Binary heating



Core collapse (Cohn 1980)

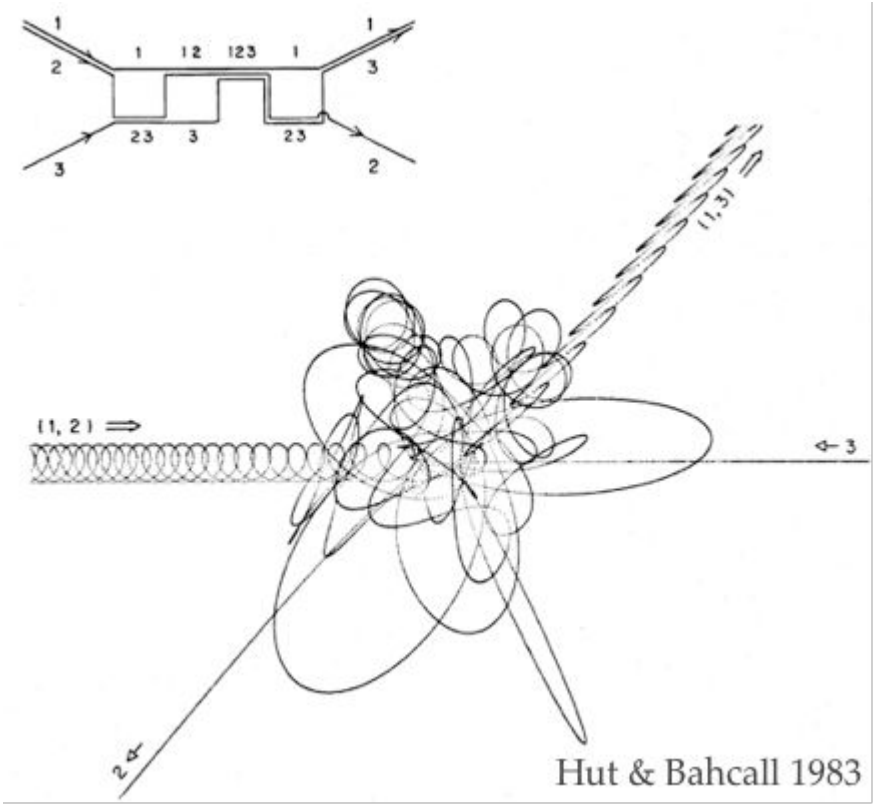
Dynamical evolution of cluster

- Mass segregation
  - Equipartition of kinetic energy
  - Dynamical friction
  - Massive components to center



http://www.astro.yale.edu/vdbosch/astro610_lecture14.pdf

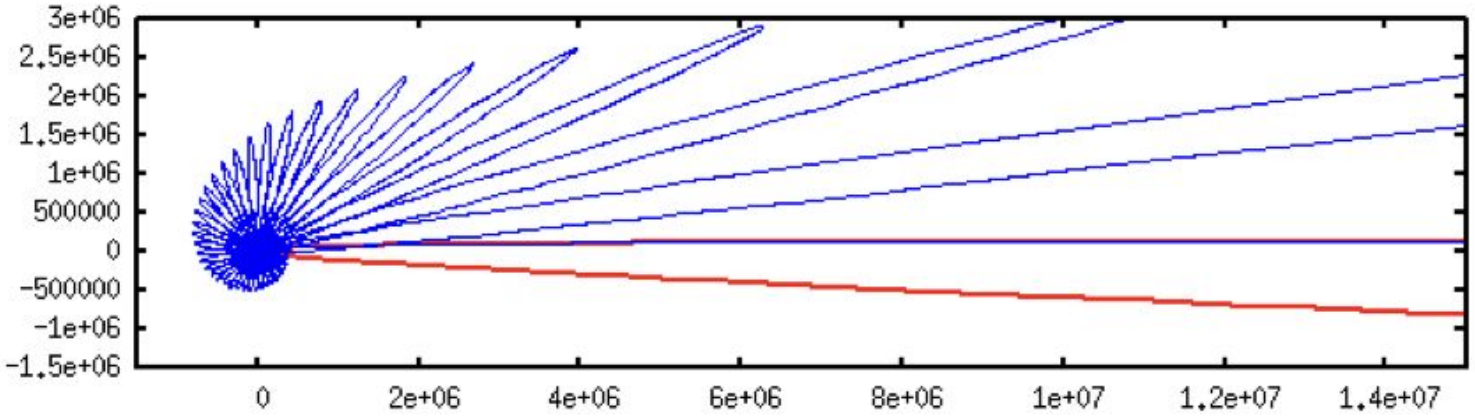
Three-body process



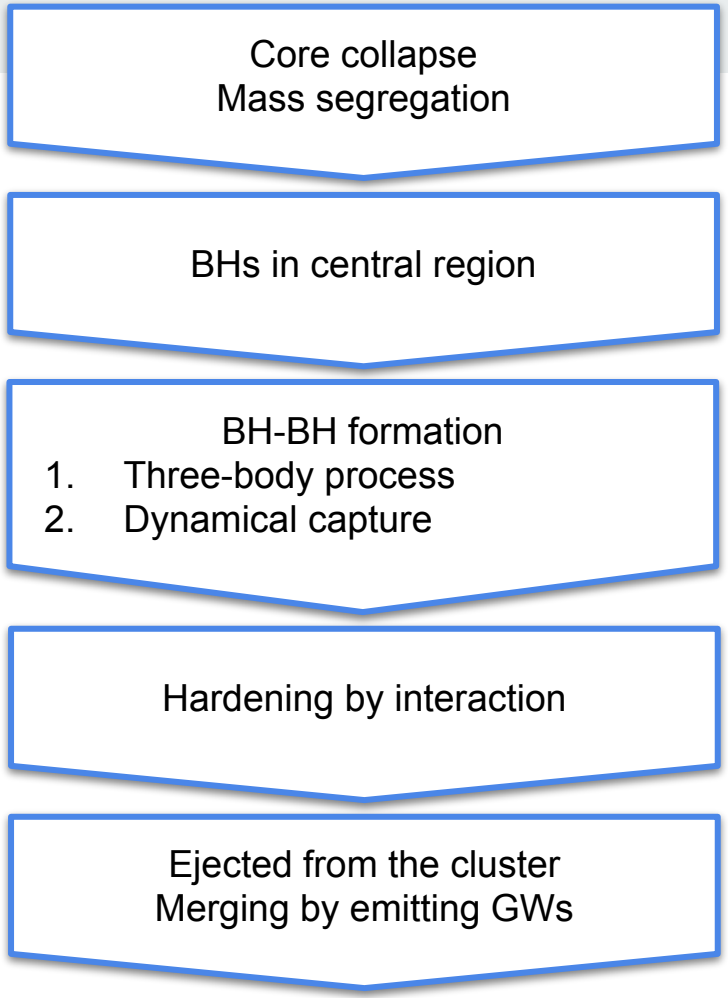
https://imgur.com/gallery/OkUBjPu

Dynamical capture

- Gravitational radiation driven capture (GR capture), or Gravitational wave capture (GW capture)
- Two body process
- Unbound orbit to bound orbit by emitting GWs
- Energy radiation > orbital energy

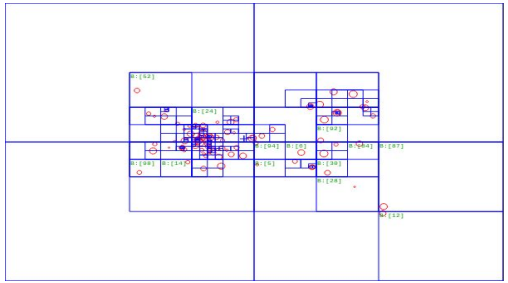


- **Dynamical evolution of cluster and binary formation**



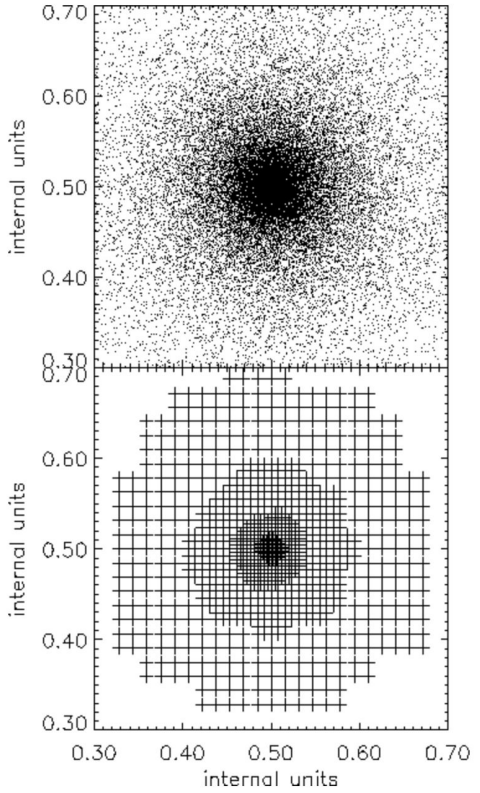
N-body

- Dynamical system of particles
  - stars, galaxies, atoms, molecules, human activities, …
- Numerical approach is required for N ≥ 3
- Direct N-body: O(N^2) forces
- Tree method: Octree (Barnes-Hut algorithm), O(N log N)
- Particle mesh: mesh of density, Cloud-in-Cell method



Barnes-Hut tree in 2D

(https://en.wikipedia.org/wiki/Barnes%E2%80%93Hut_simulation#/media/File:2D_Quad-Tree_partitioning_of_100_bodies.png)



https://astronomy.swin.edu.au/sao/guest/knebe/#mlapmref

N-body

- NBODY6
  - https://people.ast.cam.ac.uk/~sverre/web/pages/nbody.htm
  - https://github.com/nbodyx/Nbody6
- Hermite integrator: predictor-corrector
- Individual timestep, block time step
- Ahmad-Cohen neighbor scheme: regular, irregular acceleration
- Regularization
- Various initial density profile, stellar evolution, stellar binary evolution, primordial binary, …
- GPU

- Stars, NSs, and BHs are evenly mixed initially, but BHs sink toward center in early phase.
- NSs fall to the center after the BHs are exhausted.
- 30% of BHs are ejected in binary.
- BH-NS binaries are rare.

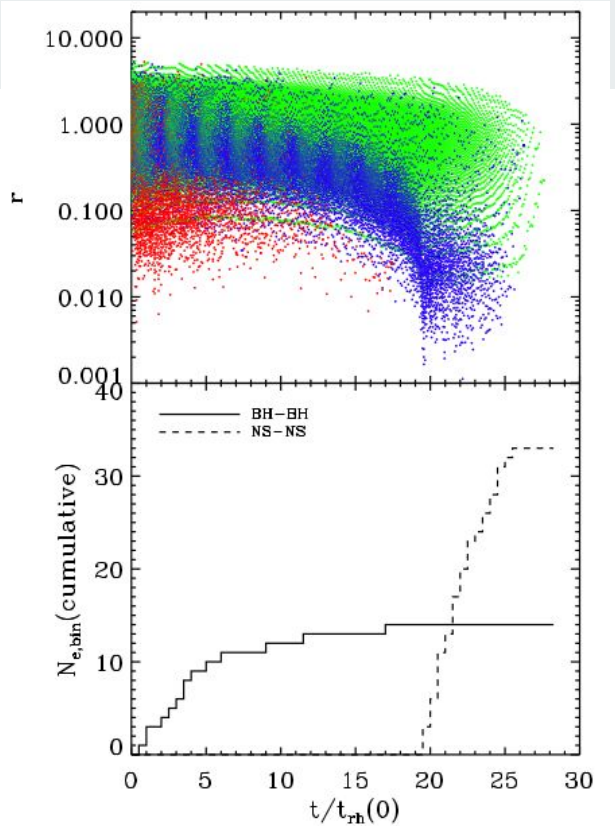
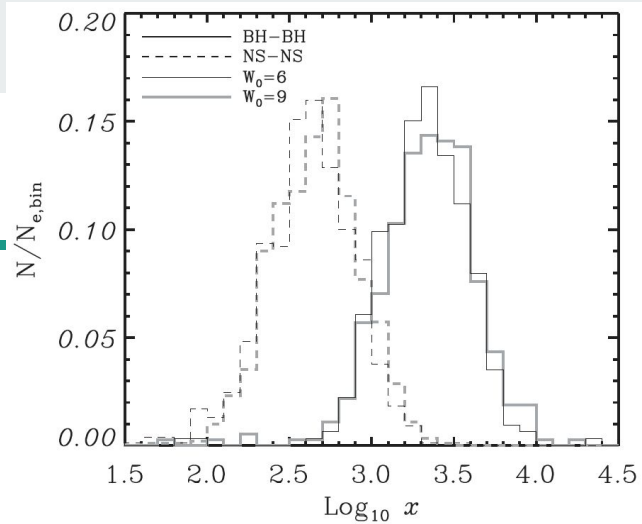
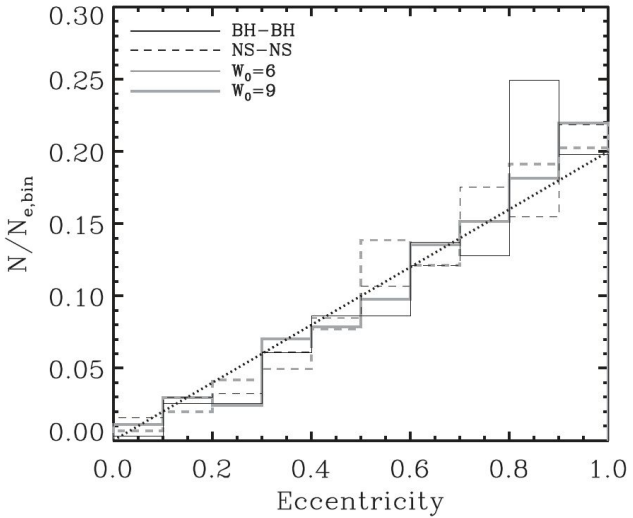


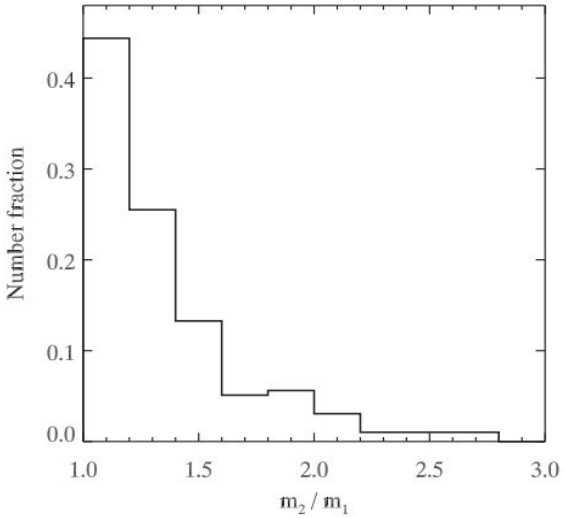
Figure 2. The upper panel: the distance of each mass component from the cluster centre is plotted against time. The total number of particles in the model is 50k, including 250 NSs and 100 BHs (model: A50kBN). Green, blue and red dots stand for the mean distance obtained from adjacent 200 ordinary stars, 4 NSs and 2 BHs, respectively. The lower panel: the cumulative number of ejected BH–BH or NS–NS binaries over time obtained from the same model.

Bae et al. (2014), Park et al. (2017)

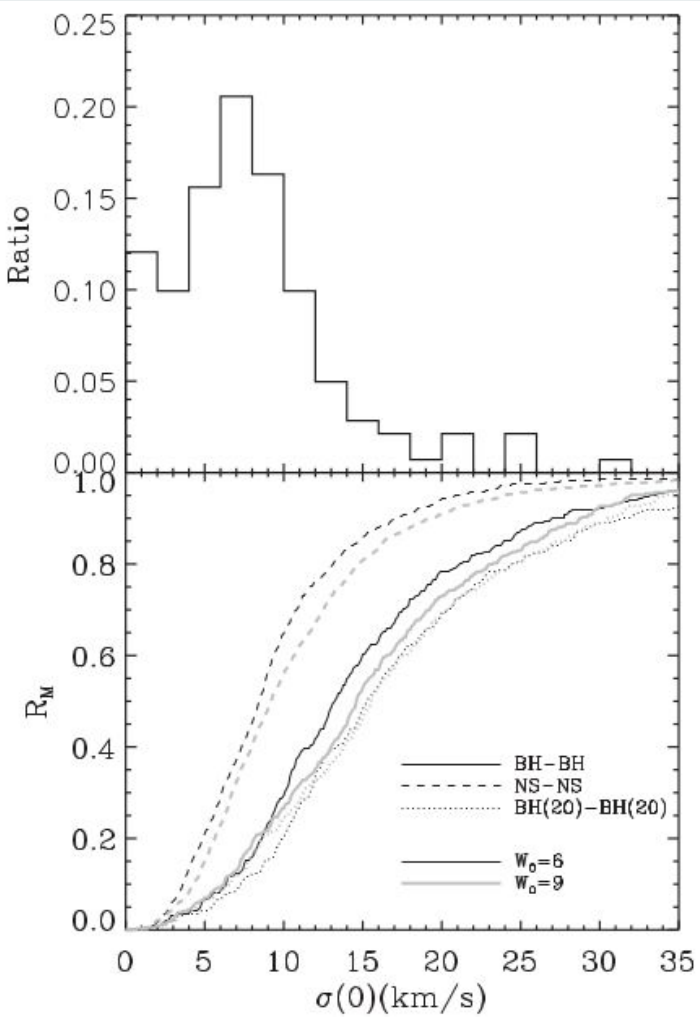




- Properties of ejected binaries
  - Tightly bound
  - High eccentricity
  - Low mass ratio



Bae et al. (2014), Park et al. (2017)



- Merger rate by considering of real globular clusters
  - Merger rate after the ejection depends on the central velocity of the cluster.
  - Merger rate of BBH is about 6.5 yr^{-1} Gpc^{-3}.
  - NS-NS binaries that are formed dynamically in the cluster are rare.

Bae et al. (2014), Park et al. (2017)

Numerical Relativity

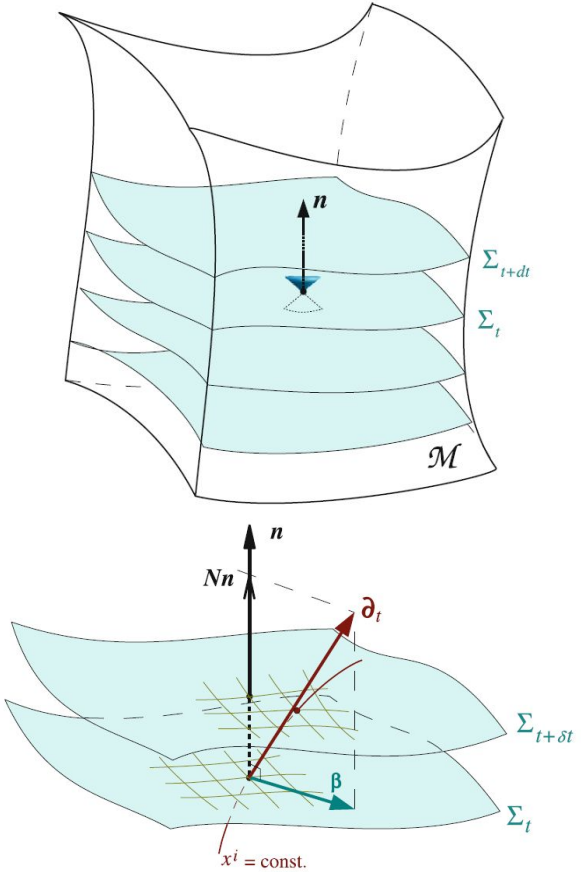
- Einstein Equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

  - Relation between geometry of spacetime and matter
- Numerical solution of Einstein equations
- Black holes, Neutron stars, Gravitational waves, …

Numerical Relativity

- 3+1 decomposition
  - 4 dimensional spacetime -> 3+1 dimension
  - Constraint equations
    - Hamiltonian constraint
    - Momentum constraint
  - Evolution equations
  - Gauge choice
    - Lapse and shift



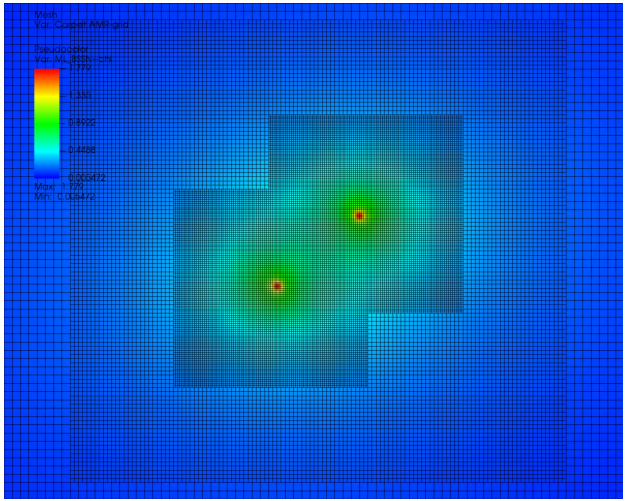
Gourgoulhon, E. 2012, 3+1 Formalism in General Relativity: Bases of Numerical Relativity, Lecture Notes in Physics, Volume 846

BBH simulation

- Time evolution schemes
  - Constrained schemes, Free evolution schemes
- BSSN scheme (Shibata and Nakamura 1995, Baumgarte and Shapiro 1999)
  - Most widely used evolution scheme
  - Long term stable evolution
- BBH simulation
  - Black hole excision (Pretorius 2005)
  - Moving puncture method (Baker et al. 2006, Campanelli et al. 2006)

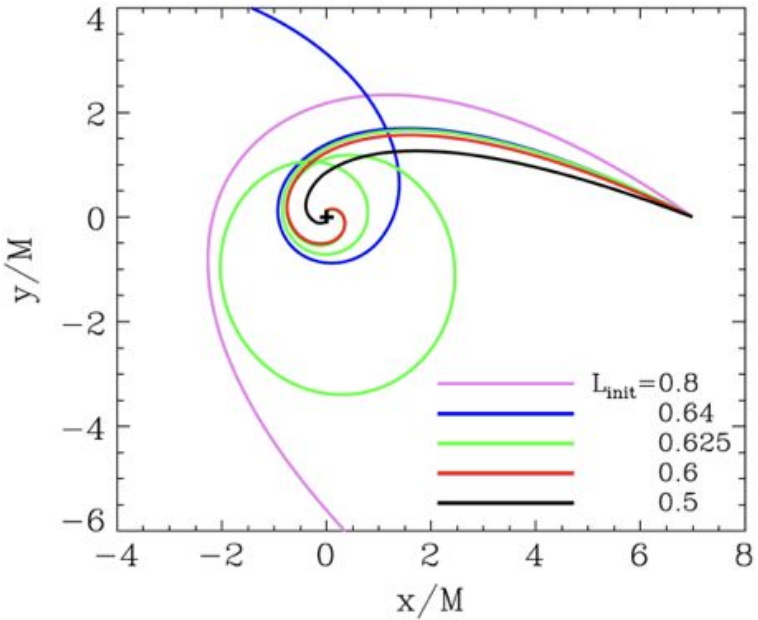
Einstein Toolkit

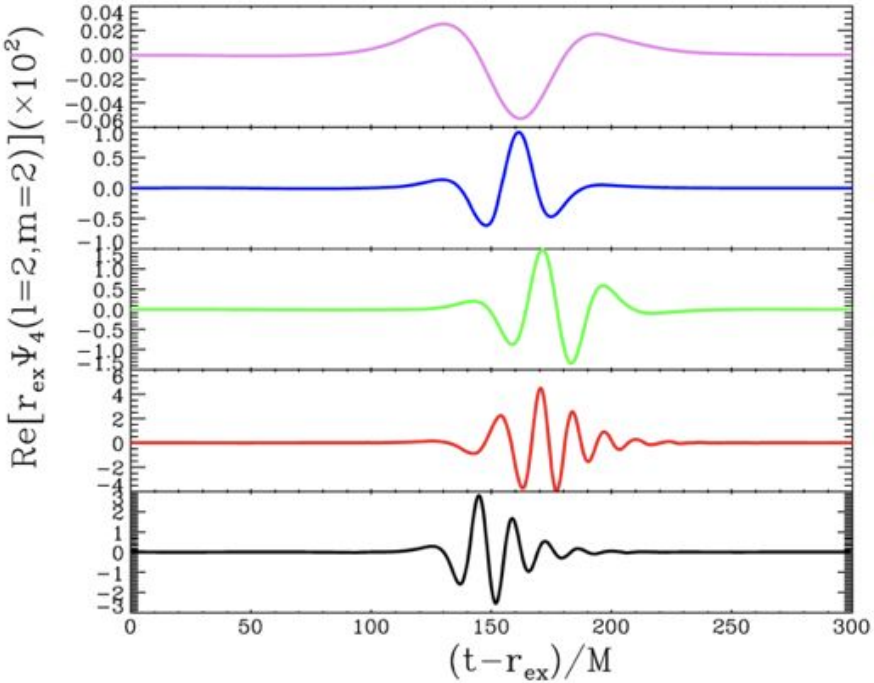
- Collection of software components and tools for simulation and analysis for general relativistic phenomena
- Free and Open source (https://einsteintoolkit.org)
- Vacuum spacetime & Relativistic hydrodynamics
- Based on Cactus code
  - Flesh (central core) + Thorns (application module)
  - Over 100 thorns
- Initial condition, Mesh refinement, Wave extraction, …
- Regular updates (Johnson, released on Dec 8th, 2021)



Orbits & Waves

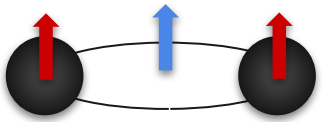
- For different angular momenta (m_1/m_2=4)



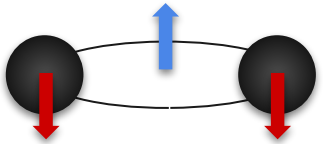


Bae et al. (2017), Bae et al. (2020)

Orbits & Waves

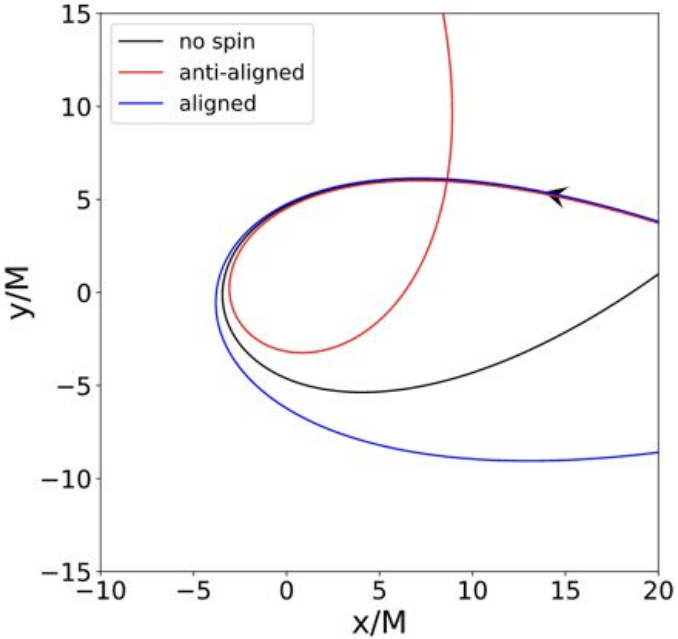


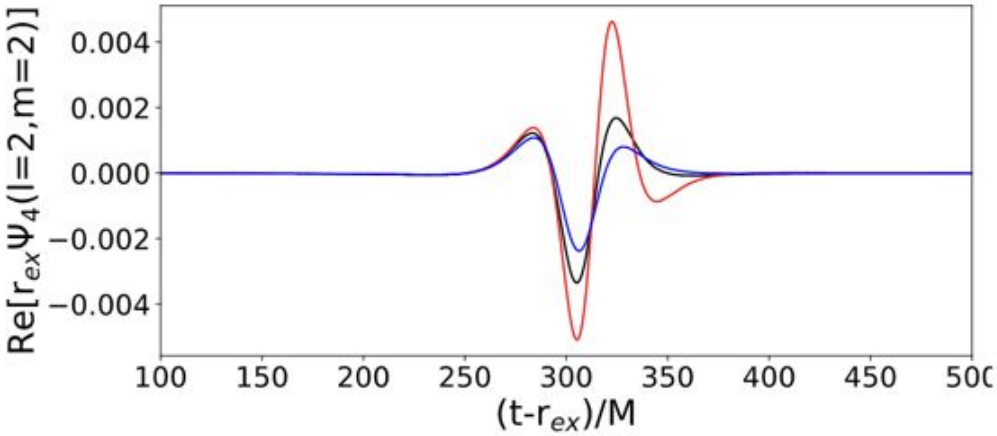
aligned



anti-aligned

- For different spins (m_1/m_2=1, L_{init}=1.11)

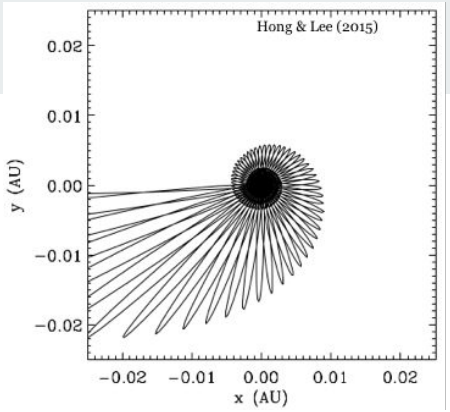




Bae et al. (2017), Bae et al. (2020)

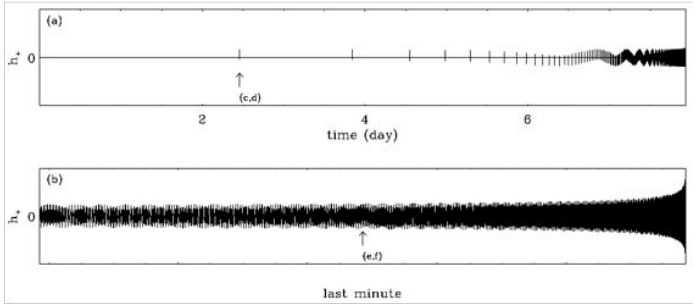
Waveform

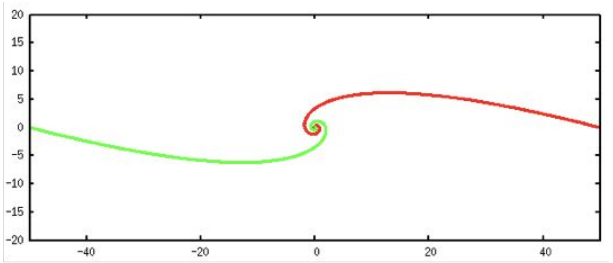
- Highly eccentric orbit
  - Some cases in dynamical capture
  - Sporadic burst
  - Merger - Ringdown without Inspiral
  - Repeated burst in a short time
  - Need new template

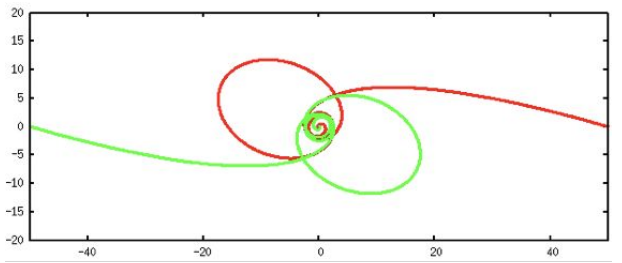


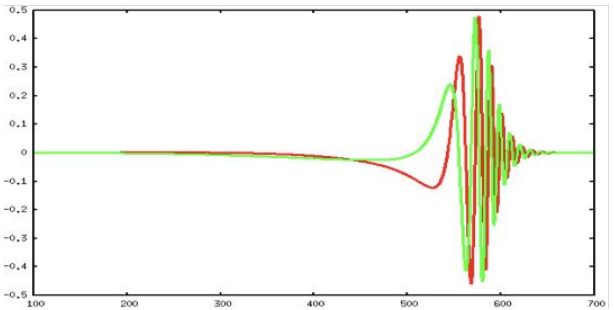
a=0.153AU
ecc=0.99989

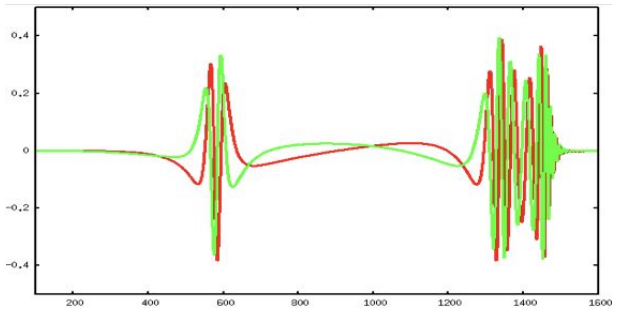
Hong and Lee (2015)











Summary

- BBHs are the main targets of GW detectors.
- BBHs can be formed dynamically in the central region of star clusters.
- Numerical methods are used in GW source studies.